Solution of Differential Equations

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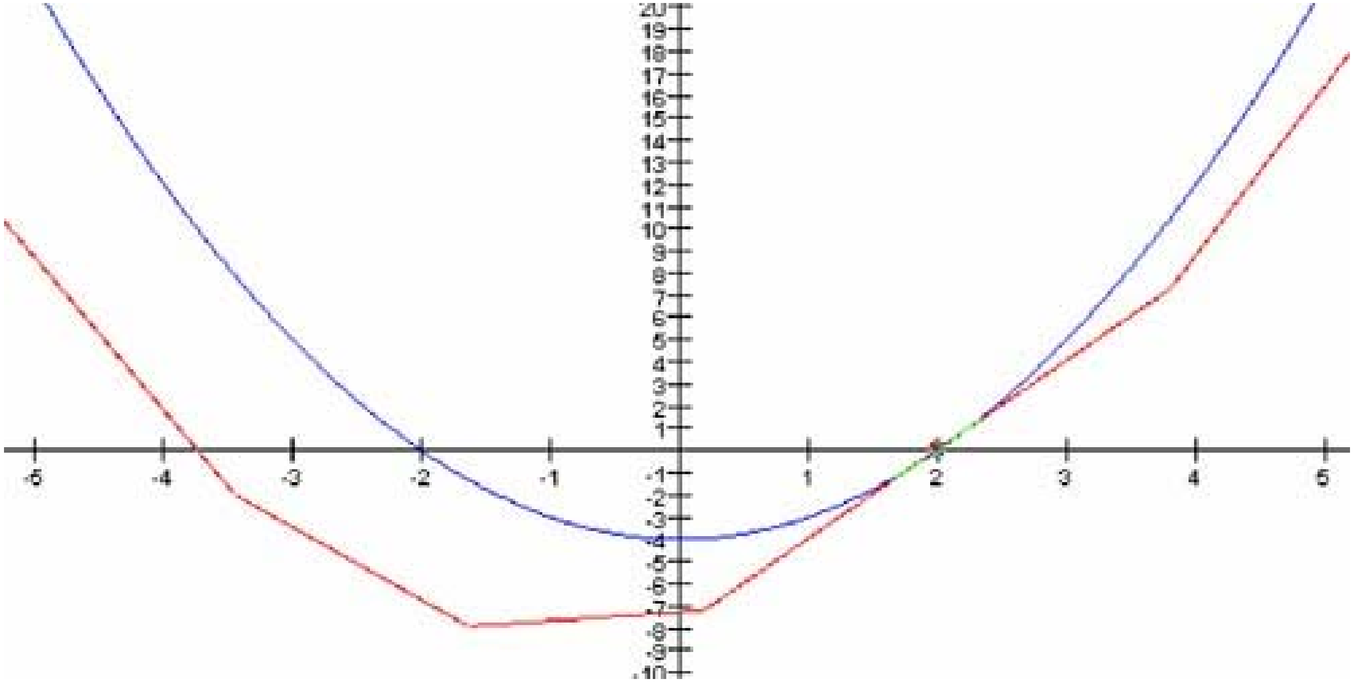
Differential Equation Examples

$y' = f(x, y)$	Initial Condition	Solution
$y' = 2x$	$y(2) = 0$	$y = x^2 - 4$
$y' = 3x^2 + 6x - 9$	$y(-4.5050397) = 0$	$y = x^3 + 3x^2 - 9x - 10$
$y' = 6x^2 - 20x + 11$	$y(0) = -5$	$y = 2x^3 - 10x^2 + 11x - 5$
$y' = 2xe^{2x} + y$	$y(0) = 1$	$y = 3e^x - 2e^{2x} + 2xe^{2x}$
$y' = 8x - 2y + 8$	$y(0) = -1$	$y = 4x - 3e^{-2x} + 2$
$y' = xe^{-2x} - 2y$	$y(0) = -0.5$	$y = \frac{x^2e^{-2x} - e^{-2x}}{2}$

Analytical Solution and Numerical Solution of Differential Equation

Differential equation : $y' = 2x$ with initial condition y(2) = 0

Analytical solution : $y = x^2 - 4$



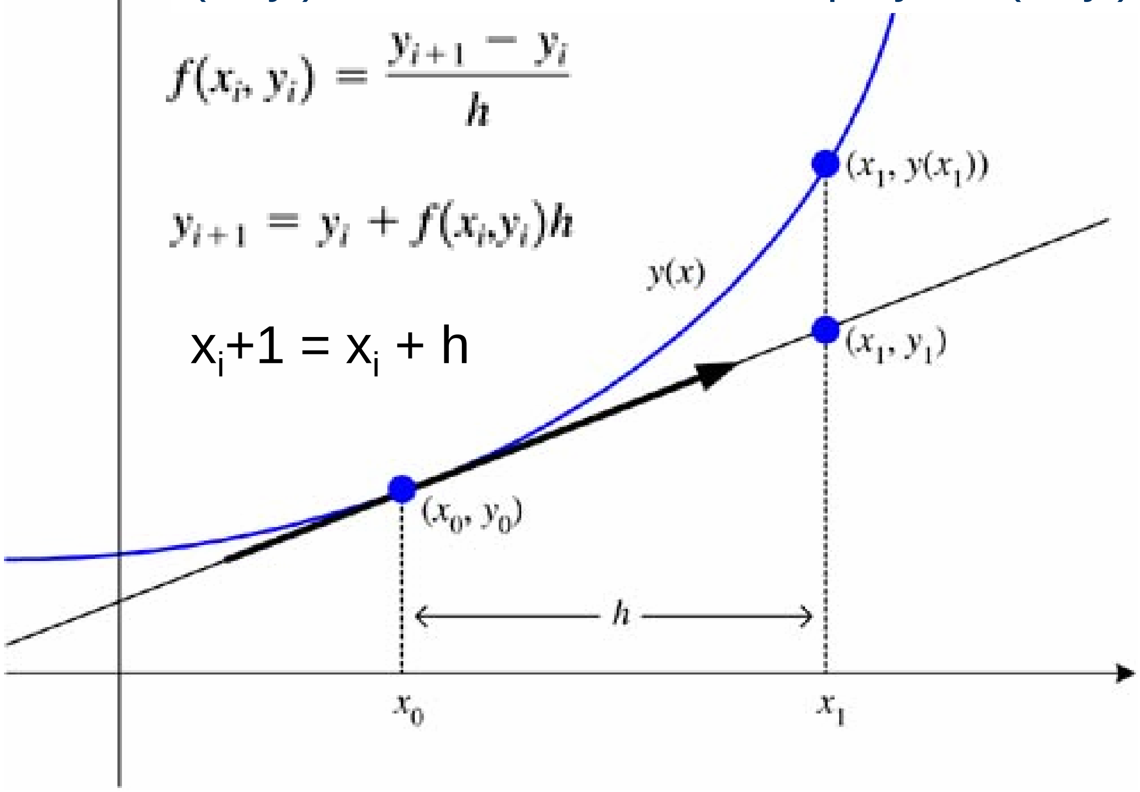
Euler’s Method

We can compute the next point (xi+1, yi+1) from the previous point (xi, yi) and the value of the slope yi' = f(xi, yi) at the previous point.

$$f(x_i, y_i) = \frac{y_{i+1} - y_i}{h}$$

$$y_{i+1} = y_i + f(x_i, y_i)h$$

$x_i+1 = x_i + h$



Algorithm

```
void Euler(float h)
{
        y += h*function(x, y);
        x += h;
}
```

Exercise for Euler's Algorithm

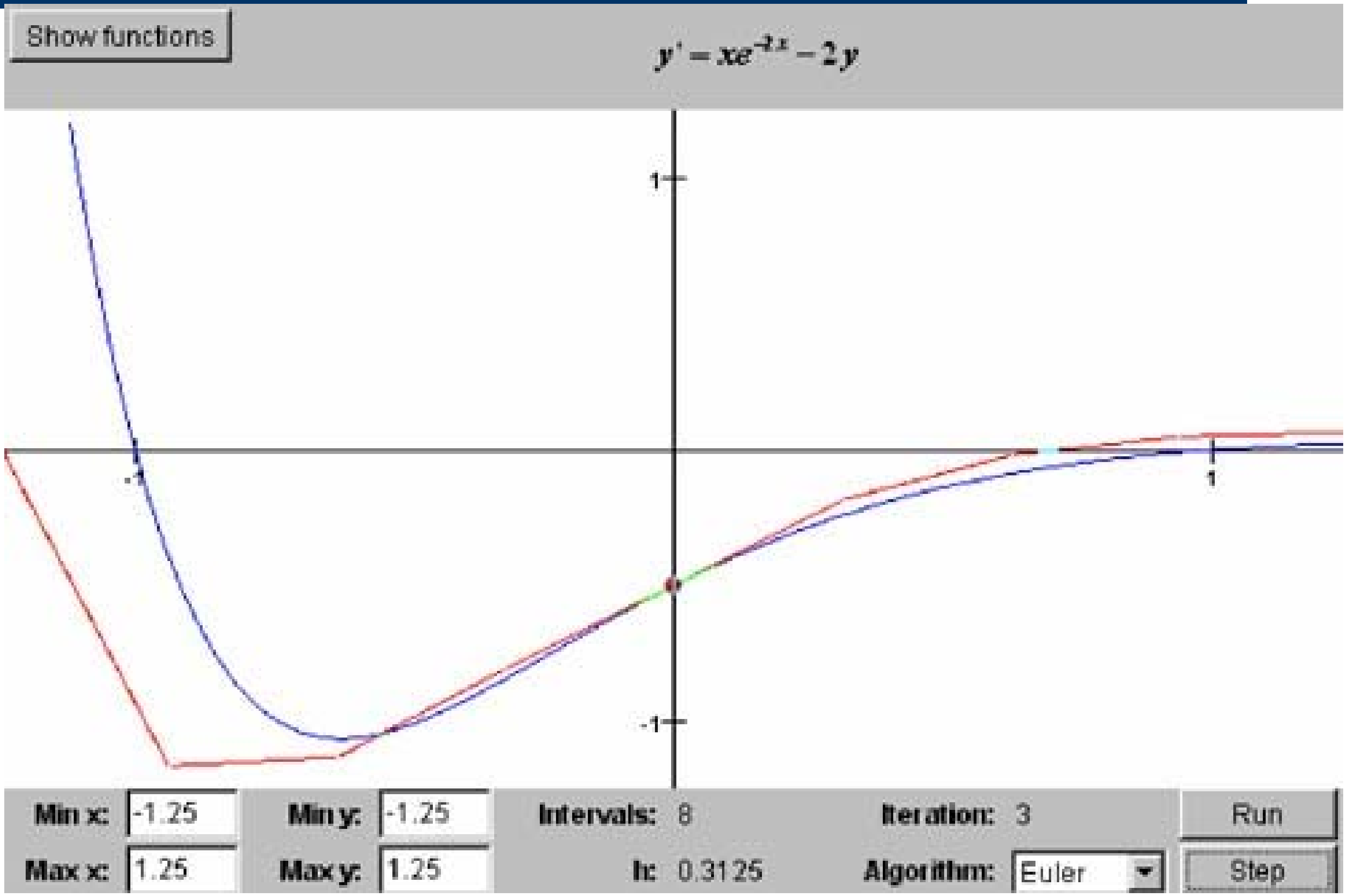
Differential equation: $y' = f(x,y) = 6x^2 - 20x + 11$

Initial condition: $y(0.0) = -5.0$

Analytical solution: $y = 2x^3 - 10x^2 + 11x - 5$

True value: $y(6.0) = 133.0$

Exercise for Euler's Algorithm



Fourth-Order Runge-Kuta Method

For y' = f(x, y)

$$k_1 = f(x_i, y_i)$$

$$k_2 = f\left(x_i + \frac{h}{2}, y_i + \frac{h}{2}k_1\right)$$

$$k_3 = f\left(x_i + \frac{h}{2}, y_i + \frac{h}{2}k_2\right)$$

$$k_4 = f(x_i + h, y_i + hk_3)$$

$$y_{i+1} = y_i + \left(\frac{k_1 + 2k_2 + 2k_3 + k_4}{6}\right)h$$

Runge-Kuta Method

```
Void RungeKuta(float h)
{
    float k1 = function(x, y);
    float k2 = function(x + h/2, y + k1*h/2);
    float k3 = function(x + h/2, y + k2*h/2);
    float k4 = function(x + h, y + k3*h);
    y += (k1 + 2*(k2 + k3) + k4)*h/6;
    x += h;
}
```

$$k_1 = f(x_i, y_i)$$

$$k_2 = f\left(x_i + \frac{h}{2}, y_i + \frac{h}{2}k_1\right)$$

$$k_3 = f\left(x_i + \frac{h}{2}, y_i + \frac{h}{2}k_2\right)$$

$$k_4 = f(x_i + h, y_i + hk_3)$$

Ex 1

Given the differential equation $y'=2xe^{2x}+y$ with $y(0)=1$, to find the solution for $x=1,2,3,4,5$

(1)Using Euler's method;

(2) Using Runge-Kuta method;

(3) Using the analytical solution $y= 3e^{x}-2e^{2x}+2xe^{2x}$.

(4)Plot above 3 curves in Excel